Chapter
Straight Lines and Pair of Straight Lines

Topic-1: Distance Formula, Section Formula, Locus, Slope of a Straight line

1 MCQs with One Correct Answer

1. Let $O(0, 0)$, $P(3, 4)$, $Q(6, 0)$ be the vertices of the triangles OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are [2007 -3 marks]

   (a) $\left(\frac{4}{3}, 3\right)$ (b) $\left(3, \frac{2}{3}\right)$ (c) $\left(3, \frac{4}{3}\right)$ (d) $\left(\frac{4}{3}, \frac{2}{3}\right)$

2. Area of the triangle formed by the line $x+y=3$ and angle bisectors of the pair of straight lines $x^2-y^2+2y=1$ is [2004S]

   (a) 2 sq. units (b) 4 sq. units
   (c) 6 sq. units (d) 8 sq. units

3. Orthocentre of triangle with vertices (0, 0), (3, 4) and (4, 0) is [2003S]

   (a) $\left(3, \frac{5}{4}\right)$ (b) (3, 12) (c) $\left(3, \frac{3}{4}\right)$ (d) (3,9)

4. The number of intergral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0,0), (0,21) and (21,0), is [2003S]

   (a) 133 (b) 190 (c) 233 (d) 105

5. A straight line through the origin O meets the parallel lines $4x+2y=9$ and $2x+y+6=0$ at points P and Q respectively. Then the point O divides the segemnt PQ in the ratio [2002S]

   (a) $1:2$ (b) $3:4$ (c) $2:1$ (d) $4:3$

6. Area of the parallelogram formed by the lines $y=mx$, $y=mx+1$, $y=nx$ and $y=nx+1$ equals [2001S]

   (a) $|m+n|/(m-n)^2$ (b) $2/|m+n|$
   (c) $1/(|m+n|)$ (d) $1/(|m-n|)$

7. The number of integer values of m, for which the x-coordinate of the point of intersection of the lines $3x+4y=9$ and $y=mx+1$ is also an integer, is [2001S]

   (a) 2 (b) 0 (c) 4 (d) 1

8. The incentre of the triangle with vertices $(1, \sqrt{3})$, (0, 0) and (2, 0) is [2000S]

   (a) $\left(1, \frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$
   (c) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (d) $\left(1, \frac{1}{\sqrt{3}}\right)$

9. The orthocentre of the triangle formed by the lines $xy=0$ and $x+y=1$ is [1995S]

   (a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{3}, \frac{1}{3}\right)$ (c) (0,0) (d) $\left(\frac{1}{4}, \frac{1}{4}\right)$

10. The locus of a variable point whose distance from (–2, 0) is 2/3 times its distance from the line $x=-\frac{9}{2}$ is [1994]

    (a) ellipse (b) parabola
    (c) hyperbola (d) none of these

11. If the sum of the distances of a point from two perpendicular lines in a plane is 1, then its locus is [1992 - 2 Marks]

    (a) square (b) circle
    (c) straight line (d) two intersecting lines

12. If $P=(1, 0)$, $Q=(-1, 0)$ and $R=(2, 0)$ are three given points, then locus of the point S satisfying the relation $SQ^2+SR^2=2SP^2$, is [1988 - 2 Marks]

    (a) a straight line parallel to x-axis
    (b) a circle passing through the origin
    (c) a circle with the centre at the origin
    (d) a straigth line parallel to y-axis.

13. The straight lines $x+y=0$, $3x+y-4=0$, $x+3y-4=0$ form a triangle which is [1983 - 1 Mark]

    (a) isosceles (b) equilateral
    (c) right angled (d) none of these

14. The point (4, 1) undergoes the following three transformations successively. [1980]
(i) Reflection about the line $y = x$.
(ii) Translation through a distance 2 units along the positive direction of x-axis.
(iii) Rotation through an angle $\pi/4$ about the origin in the counter clockwise direction.
Then the final position of the point is given by the coordinates.
(a) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (b) $(-\sqrt{2}, 7\sqrt{2})$
(c) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (d) $(\sqrt{2}, 7\sqrt{2})$

4 Fill in the Blanks

15. The vertices of a triangle are $A(-1, -7)$, $B(5, 1)$ and $C(1, 4)$. The equation of the bisector of the angle $\angle ABC$ is .... [1993 - 2 Marks]
16. The orthocentre of the triangle formed by the lines $x + y = 1$, $2x + 3y = 6$ and $4x - y + 4 = 0$ lies in quadrant number .... [1985 - 2 Marks]
17. Given the points $A(0, 4)$ and $B(0, -4)$, the equation of the locus of the point $P(x, y)$ such that $|AP - BP| = 6$ is .... [1983 - 1 Mark]
18. The area enclosed within the curve $|x| + |y| = 1$ is .... [1981 - 2 Marks]

5 True / False

19. The straight line $5x + 4y = 0$ passes through the point of intersection of the straight lines $x + 2y - 10 = 0$ and $2x + y + 5 = 0$. [1983 - 1 Mark]

6 MCQs with One or More than One Correct Answer

20. The diagonals of a parallelogram $PQRS$ are along the lines $x + 3y = 4$ and $6x - 2y = 7$. Then $PQRS$ must be a. [1998 - 2 Marks]
(a) rectangle (b) square
(c) cyclic quadrilateral (d) rhombus
21. If ($P(1, 2)$, $Q(4, 6)$, $R(5, 7)$ and $S(a, b)$ are the vertices of a parallelogram $PQRS$, then [1998 - 2 Marks]
(a) $a = 2, b = 4$ (b) $a = 3, b = 4$
(c) $a = 2, b = 3$ (d) $a = 3, b = 5$
22. All points lying inside the triangle formed by the points (1, 3), (5, 0) and (–1, 2) satisfy [1986 - 2 Marks]
(a) $3x + 2y \geq 0$ (b) $2x + y - 13 \geq 0$
(c) $2x - 3y - 12 \leq 0$ (d) $-2x + y \geq 0$
(e) none of these.
23. The points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30) are vertices of
(a) an obtuse angled triangle [1986 - 2 Marks]
(b) an acute angled triangle
(c) a right angled triangle
(d) an isosceles triangle
(e) none of these.

10 Subjective Problems

24. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through $P(h, k)$ with the lines $y = x$ and $x + y = 2$ is $4h^2$. Find the locus of the point P. [2005 - 2 Marks]
25. A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinate axes at points P and Q. Find the absolute minimum value of $OP + OQ$, as L varies, where O is the origin. [2002 - 5 Marks]
26. Let ABC and PQR be any two triangles in the same plane. Assume that the prependiculars from the points A, B, C to the sides QR, RP, PQ respectively are concurrent. Using vector methods or otherwise, prove that the prependiculars from P, Q, R to BC, CA, AB respectively are also concurrent. [2000 - 10 Marks]
27. Using co-ordinate geometry, prove that the three altitudes of any triangle are concurrent. [1998 - 8 Marks]
28. A rectangle $PQRS$ has its side PQ parallel to the line $y = mx$ and vertices P, Q and S on the lines $y = a, x = b$ and $x = -b$, respectively. Find the locus of the vertex R. [1996 - 2 Marks]
29. Tangent at a point P_1 {other than (0, 0)} on the curve $y = x^3$ meets the curve again at P_2. The tangent at P_2 meets the curve at P_3, and so on. Show that the abscissae of $P_1, P_2, P_3 .... P_n$, form a G.P. Also find the ratio.
[area $(\Delta P_1 P_2 P_3)$] / [area $(\Delta P_2 P_3 P_4)$] [1993 - 5 Marks]
30. Determine all values of α for which the point (α, α^2) lies inside the triangle formed by the lines [1992 - 6 Marks]
$2x + 3y - 1 = 0$
$x + 2y - 3 = 0$
$5x - 6y - 1 = 0$
31. A line cuts the x-axis at $A(7, 0)$ and the y-axis at $B(0, -5)$. A variable line PQ is drawn perpendicular to AB cutting the x-axis in P and the y-axis in Q. If AQ and BP intersect at R, find the locus of R. [1990 - 4 Marks]
32. Two sides of a rhombus $ABCD$ are parallel to the lines $y = x + 2$ and $y = 7x + 3$. If the diagonals of the rhombus intersect at the point (1, 2) and the vertex A is on the y-axis, find possible co-ordinates of A. [1985 - 5 Marks]

33. One of the diameters of the circle circumscribing the rectangle $ABCD$ is $4y = x + 7$. If A and B are the points (–3, 4) and (5, 4) respectively, then find the area of rectangle. **[1985 - 3 Marks]**

34. The coordinates of A, B, C are (6, 3), (–3, 5), (4, – 2) respectively, and P is any point (x, y). Show that the ratio of the area of the triangles ΔPBC and ΔABC is $\left|\frac{x+y-2}{7}\right|$ **[1983 - 2 Marks]**

35. The vertices of a triangle are $[at_1t_2, a(t_1 + t_2)]$, $[at_2t_3, a(t_2+t_3)]$, $[at_3t_1, a(t_3+t_1)]$. Find the orthocentre of the triangle. **[1983 - 3 Marks]**

Topic-2: Various Forms of Equation of a Line

1 MCQs with One Correct Answer

1. A straight line L through the point (3, –2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is **[2011]**
   (a) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$
   (b) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$
   (c) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$
   (d) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

2. Let PS be the median of the triangle with vertices $P(2, 2)$, $Q(6, -1)$ and $R(7, 3)$. The equation of the line passing through (1,–1) and parallel to PS is **[2000S]**
   (a) $2x - 9y - 7 = 0$ (b) $2x - 9y - 11 = 0$
   (c) $2x + 9y - 11 = 0$ (d) $2x + 9y + 7 = 0$

3. The equations to a pair of opposite sides of parallelogram are $x^2 - 5x + 6 = 0$ and $y^2 - 6y + 5 = 0$, the equations to its diagonals are **[1994]**
   (a) $x + 4y = 13, y = 4x - 7$ (b) $4x + y = 13, 4y = x - 7$
   (c) $4x + y = 13, y = 4x - 7$ (d) $y - 4x = 13, y + 4x = 7$

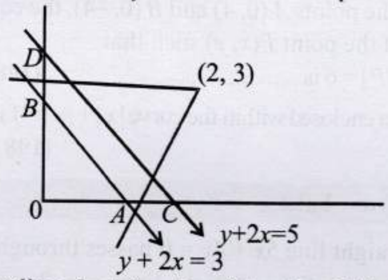
6 MCQs with One or More than One Correct Answer

4. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1? **[1999 - 3 Marks]**
   (a) $x + y = 0$ (b) $x - y = 0$
   (c) $x + 7y = 0$ (d) $x - 7y = 0$

10 Subjective Problems

5. For points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ of the co-ordinate plane, a new distance $d(P, Q)$ is defined by $d(P, Q) = |x_1 - x_2| + |y_1 - y_2|$. Let $O = (0, 0)$ and $A = (3, 2)$. Prove that the set of points in the first quadrant which are equidistant (with respect to the new distance) from O and A consists of the union of a line segment of finite length and an infinite ray. Sketch this set in a labelled diagram. **[2000 - 10 Marks]**

6. Find the equation of the line passing through the point (2, 3) and making intercept of length 2 units between the lines $y + 2x = 3$ and $y + 2x = 5$. **[1991 - 4 Marks]**



7. Straight lines $3x + 4y = 5$ and $4x - 3y = 15$ intersect at the point A. Points B and C are chosen on these two lines such that $AB = AC$. Determine the possible equations of the line BC passing through the point (1, 2). **[1990 - 4 Marks]**

8. Lines $L_1 \equiv ax + by + c = 0$ and $L_2 \equiv lx + my + n = 0$ intersect at the point P and make an angle θ with each other. Find the equation of a line L different from L_2 which passes through P and makes the same angle θ with L_1. **[1988 - 5 Marks]**

9. Two equal sides of an isosceles triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point (1, –10). Determine the equation of the third side. **[1984 - 4 Marks]**

10. A straight line L is perpendicular to the line $5x - y = 1$. The area of the triangle formed by the line L and the coordinate axes is 5. Find the equation of the line L. **[1980]**

11. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (–3, 1) and (1, 1). Find the equations of the other three sides. **[1978]**

Topic-3: Distance Between two Lines, Angle Between two Lines and Bisector of the Angle Between the two Lines

1 MCQs with One Correct Answer

1. Let $P=(-1, 0)$, $Q=(0, 0)$ and $R=(3, 3\sqrt{3})$ be three points. Then the equation of the bisector of the angle PQR is [2002S]

(a) $\frac{\sqrt{3}}{2}x+y=0$ (b) $x+\sqrt{3}y=0$

(c) $\sqrt{3}x+y=0$ (d) $x+\frac{\sqrt{3}}{2}y=0$

2. Let $0<\alpha<\frac{\pi}{2}$ be fixed angle. If

$P=(\cos\theta, \sin\theta)$ and $Q=(\cos(\alpha-\theta), \sin(\alpha-\theta))$,

then Q is obtained from P by [2002S]

(a) clockwise rotation around origin through an angle α
(b) anticlockwise rotation around origin through an angle α
(c) reflection in the line through origin with slope $\tan\alpha$
(d) reflection in the line through origin with slope $\tan(\alpha 2)$

3. Line L has intercepts a and b on the coordinate axes. When the axes are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q, then [1990 - 2 Marks]

(a) $a^2+b^2=p^2+q^2$ (b) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$

(c) $a^2+p^2=b^2+q^2$ (d) $\frac{1}{a^2}+\frac{1}{p^2}=\frac{1}{b^2}+\frac{1}{q^2}$

4. The points $(-a,-b)$, $(0, 0)$, (a, b) and (a^2, ab) are : [1979]

(a) Collinear
(b) Vertices of a parallelogram
(c) Vertices of a rectangle
(d) None of these

2 Integer Value Answer/ Non-Negative Integer

5. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distance of the point P from the lines $x-y=0$ and $x+y=0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P)+d_2(P) \le 4$, is [Adv. 2014]

3 Numeric/ New Stem Based Questions

Consider the lines L_1 and L_2 defined by

$L_1: x\sqrt{2}+y-1=0$ and $L_2: x\sqrt{2}-y+1=0$

For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ_2. The line $y=2x+1$ meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R' and S'. Let D be the **square** of the distance between R' and S'.

6. The value of λ^2 is _____. [Adv. 2021]
7. The value of D is _____. [Adv. 2021]

4 Fill in the Blanks

8. Let the algebraic sum of the perpendicular distances from the points (2, 0), (0, 2) and (1, 1) to a variable straight line be zero; then the line passes through a fixed point whose cordinates are .... [1991 - 2 Marks]

9. If a, b and c are in A.P., then the straight line $ax+by+c=0$ will always pass through a fixed point whose coordinates are ________ [1984 - 2 Marks]

10. The set of lines $ax+by+c=0$, where $3a+2b+4c=0$ is concurrent at the point .... [1982 - 2 Marks]

5 True / False

11. The straight line $5x+4y=0$ passes through the point of intersection of the straight lines $x+2y-10=0$ and $2x+y+5=0$. [1983 - 1 Mark]

6 MCQs with One or More than One Correct Answer

12. For $a>b>c>0$, the distance between (1, 1) and the point of intersection of the lines $ax+by+c=0$ and $bx+ay+c=0$ is less than $2\sqrt{2}$. Then [Adv. 2013]

(a) $a+b-c>0$ (b) $a-b+c<0$

(c) $a-b+c>0$ (d) $a+b-c<0$

13. Three lines $px+qy+r=0$, $qx+ry+p=0$ and $rx+py+q=0$ are concurrent if [1985 - 2 Marks]

(a) $p+q+r=0$

(b) $p^2+q^2+r^2=qr+rp+pq$

(c) $p^3+q^3+r^3=3pqr$

(d) none of these.

9 Assertion and Reason/Statement Type Questions

14. Lines $L_1: y-x=0$ and $L_2: 2x+y=0$ intersect the line $L_3: y+2=0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

STATEMENT-1 : The ratio $PR : RQ$ equals $2\sqrt{2} : \sqrt{5}$.

STATEMENT-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

[2007 - 3 marks]

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True.

10 Subjective Problems

15. Let ABC be a triangle with $AB = AC$. If D is the midpoint of BC, E is the foot of the perpendicular drawn from D to AC and F the mid-point of DE, prove that AF is perpendicular to BE. **[1989 - 5 Marks]**

Topic-4: Pair of Straight lines

1 MCQs with One Correct Answer

1. Let PQR be a right angled isosceles triangle, right angled at $P(2, 1)$. If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is **[1999 - 2 Marks]**
   (a) $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$
   (b) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$
   (c) $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$
   (d) $3x^2 - 3y^2 - 8xy - 10x - 15y - 20 = 0$

10 Subjective Problems

2. Show that all chords of the curve $3x^2 - y^2 - 2x + 4y = 0$, which subtend a right angle at the origin, pass through a fixed point. Find the coordinates of the point. **[1991 - 4 Marks]**

Answer Key

Topic-1 : Distance Formula, Section Formula, Locus, Slope of a Straight Line

1. (c)	2. (a)	3. (c)	4. (b)	5. (b)	6. (d)	7. (a)	8. (d)	9. (c)	10. (a)
11. (a)	12. (d)	13. (a)	14. (c)	15. $(x-7y+2=0)$	16. (First)	17. $\frac{y^2}{9}-\frac{x^2}{7}=1$	18. (2)		
19. (True)	20. (d)	21. (c)	22. (a, c)	23. (e)					

Topic-2 : Various Forms of Equation of a Line

1. (b)	2. (d)	3. (c)	4. (b, c)

Topic - 3 : Distance Between two Lines, Angle Between two Lines and Bisector of the Angle Between the two Lines

1. (c)	2. (d)	3. (b)	4. (a)	5. (6)	6. (9)	7. (77.14)	8. (1, 1)	9. (1, –2)
10. $\left(\frac{3}{4}, \frac{1}{2}\right)$	11. (True)	12. (a)	13. (a, c)	14. (c)				

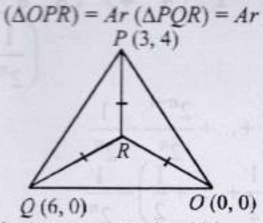
Topic-4 : Pair of Straight Lines

1. (b)

Hints & Solutions

Topic-1: Distance Formula, Section Formula, Locus, Equation of Locus, Slope of a Straight Line

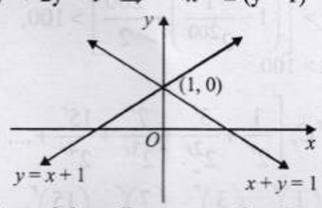
1. **(c)** $\because$ $Ar(\Delta OPR) = Ar(\Delta PQR) = Ar(\Delta OQR)$



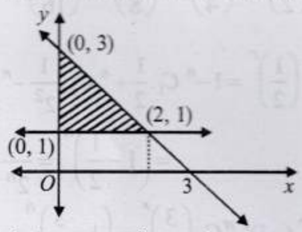
$\therefore$ By simply geometry, R should be the centroid of ΔPQO

$\Rightarrow$ co-ordinate of $R = \left(\frac{3+6+0}{3}, \frac{4+0+0}{3}\right) = \left(3, \frac{4}{3}\right)$

2. **(a)** $x^2 - y^2 + 2y = 1 \Rightarrow x = \pm(y-1)$



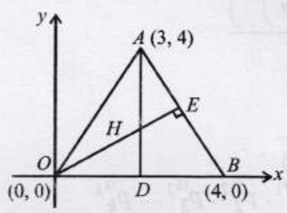
Bisectors of above lines are x = 0 and y = 1.



$\therefore$ Area between x = 0, y = 1 and x + y = 3 is the shaded region shown in figure.

$\therefore$ Area $= \frac{1}{2} \times 2 \times 2 = 2$ sq. units.

3. **(c)** We know that point of intersection of altitudes of a triangle is the orthocentre of the triangle.



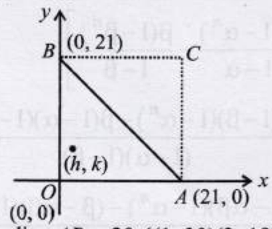
Equation of altitude AD
i.e., line parallel to y-axis through (3, 4) is
x = 3 .... (i)

Now, equation of $OE \perp AB$ is

$y = \frac{3-4}{4-0}x \Rightarrow y = x/4$ .... (ii)

Solving (i) and (ii), we get orthocentre as (3, 3/4).

4. **(b)** Total number of points within the square $OACB$ = 20 × 20 = 400



Points on line AB = 20 {(1, 20)(2, 19), (3, 18) .... (10, 11)(11, 10)... (20, 1)}

$\therefore$ Points within ΔOBC and ΔABC = 400 – 20 = 380

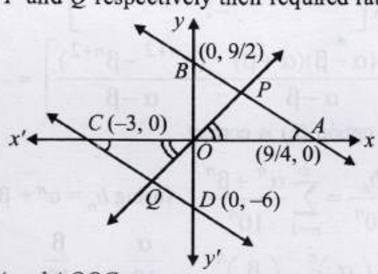
By symmetry, points within ΔOAB $= \frac{380}{2} = 190$

5. **(b)** The given lines are

$2x + y = 9/2$ .... (i)

and $2x + y = -6$ .... (ii)

Signs of constants on R.H.S. show that two lines lie on opposite sides of origin. Let a line through origin meets these lines in P and Q respectively then required ratio is $OP : OQ$



In ΔOPA and ΔOQC,

$\angle POA = \angle QOC$ (ver. opp. angles)

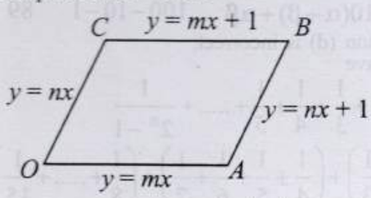
$\angle PAO = \angle OCQ$ (alt. int. angles)

$\therefore$ $\Delta OPA \sim \Delta OQC$ (By AA similarly)

$\therefore$ $\frac{OP}{OQ} = \frac{OA}{OC} = \frac{9/4}{3} = \frac{3}{4}$

$\therefore$ Required ratio = 3 : 4.

6. **(d)**



The vertices, $O(0,0), A\left(\frac{1}{m-n}, \frac{m}{m-n}\right), B(0,1)$

Area (parallelogram $OABC$) = 2 area (ΔOAB)

$= 2 \times \frac{1}{2}\left|\left[0\left(\frac{m}{m-n} - 1\right) + \frac{1}{m-n}(1-0) + 0\left(0 - \frac{m}{m-n}\right)\right]\right|$

$= \frac{1}{|m-n|}$

7. (a) $3x + 4y = 9$ and $y = mx + 1$ are two lines.
On equating the value of y from both equations to get the x co-ordinate of the point of intersection,
$3x + 4(mx + 1) = 9 \Rightarrow (3 + 4m)x = 5$

$\Rightarrow \quad x = \dfrac{5}{3+4m}$

For x to be an integer $3 + 4m$ should be a divisor of 5 i.e., $1, -1, 5$ or -5.
$3 + 4m = 1 \quad \Rightarrow m = -1/2$ (not integer)
$3 + 4m = -1 \Rightarrow m = -1$ (integer)
$3 + 4m = 5 \quad \Rightarrow m = 1/2$ (not an integer)
$3 + 4m = -5 \Rightarrow m = -2$ (integer)
$\therefore$ There are 2 integral values of m.

8. (d) Let $(1, \sqrt{3})$, $(0, 0)$ and $(2, 0)$ are the coordinates of vertices A, O, B of ΔABC.
$\therefore$ AO = OB = AB. So, it is an equilateral triangle and the incentre coincides with centroid.

$\therefore$ Incentre $= \left(\dfrac{0+1+2}{3}, \dfrac{0+0+\sqrt{3}}{3}\right) = \left(1, \dfrac{1}{\sqrt{3}}\right)$

9. (c) The lines by which triangle is formed are $x = 0$, $y = 0$ and $x + y = 1$.
Clearly, the triangle is right angled and we know that in a right angled triangle orthocentre coincides with the vertex at which right angle is formed.
$\therefore$ Orthocentre is $(0, 0)$.

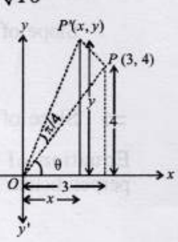
10. (a) Let variable point is P and fixed point $S(-2, 0)$, then $PS = \dfrac{2}{3}PM$ where PM is the perpendicular distance of point P from given line $x = -9/2$
$\therefore$ By definition of ellipse, P describes an ellipse with eccentricity $e = \dfrac{2}{3} < 1$

11. (a) Let the two perpendicular lines be the co-ordinate axes. Let (x, y) be the point sum of whose distances from two axes is 1 then we must have
$|x| + |y| = 1 \quad$ or $\quad \pm x \pm y = 1$
These are the four lines
$x + y = 1, x - y = 1, -x + y = 1, -x - y = 1$
Any two adjacent sides are perpendicular to each other. Also each line is equidistant from origin. Therefore figure formed i.e., locus of the point is a square.

12. (d) Given :
$P = (1, 0), Q = (-1, 0), R = (2, 0)$
Let $\quad S = (x, y)$
Now, $SQ^2 + SR^2 = 2SP^2$
$\Rightarrow (x + 1)^2 + y^2 + (x - 2)^2 + y^2 = 2[(x - 1)^2 + y^2]$
$\Rightarrow 2x^2 + 2y^2 - 2x + 5 = 2x^2 + 2y^2 - 4x + 2$
$\Rightarrow 2x + 3 = 0 \Rightarrow x = -3/2$, which is the locus of point S.
This locus is a straight line parallel to y-axis.

13. (a) Solving the given equations of lines pairwise, we get the vertices of the triangle as
$A(-2, 2)$, $B(2, -2)$ and $C(1, 1)$
Then $AB = \sqrt{16+16} = 4\sqrt{2}$,
$BC = \sqrt{1+9} = \sqrt{10}$ and $CA = \sqrt{9+1} = \sqrt{10}$
$\therefore$ The triangle is isosceles.

14. (c) Reflection about the line $y = x$, changes the point $(4, 1)$ to $(1, 4)$.
On translation of $(1, 4)$ through a distance of 2 units along positive direction of x-axis, the point becomes $(1 + 2, 4)$, i.e., $(3, 4)$.
On rotation about origin through an angle $\pi/4$ the point P takes the position P' such that $OP = OP'$



Also $OP = 5 = OP'$ and $\cos\theta = \dfrac{3}{5}$, $\sin\theta = \dfrac{4}{5}$

Now, $x = OP'\cos\left(\dfrac{\pi}{4}+\theta\right)$

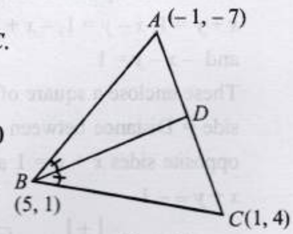
$= 5\left(\cos\dfrac{\pi}{4}\cos\theta - \sin\dfrac{\pi}{4}\sin\theta\right) = 5\left(\dfrac{3}{5\sqrt{2}} - \dfrac{4}{5\sqrt{2}}\right) = -\dfrac{1}{\sqrt{2}}$

$y = OP'\sin\left(\dfrac{\pi}{4}+\theta\right) = 5\left(\sin\dfrac{\pi}{4}\cos\theta + \cos\dfrac{\pi}{4}\sin\theta\right)$

$= 5\left(\dfrac{3}{5\sqrt{2}} + \dfrac{4}{5\sqrt{2}}\right) = \dfrac{7}{\sqrt{2}}$

$\therefore \quad P' = \left(-\dfrac{1}{\sqrt{2}}, \dfrac{7}{\sqrt{2}}\right)$

15. Let BD be the bisector of $\angle ABC$.
Then $AD : DC = AB : BC$
And



$AB = \sqrt{(5+1)^2 + (1+7)^2} = 10$
$BC = \sqrt{(5-1)^2 + (1-4)^2} = 5$
$\therefore \quad AD : DC = 2 : 1$

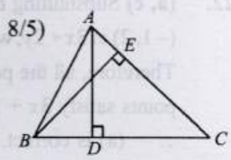
$\therefore$ By section formula coordinate of D is $\left(\dfrac{1}{3}, \dfrac{1}{3}\right)$

Therefore equation of BD is

$y - 1 = \dfrac{1/3 - 1}{1/3 - 5}(x - 5) \Rightarrow y - 1 = \dfrac{-2/3}{-14/3}(x - 5)$

$\Rightarrow x - 7y + 2 = 0$

16. The equations of sides of triangle ABC are
$AB \quad : x + y = 1$
$BC \quad : 2x + 3y = 6$
$CA \quad : 4x - y = -4$
Solving these equations pairwise, we get the vertices of the triangle as
$A(-3/5, 8/5)$, $B(-3, 4)$ and $C(-3/7, 16/7)$.
Let $AD \perp BC$ as shown in the figure. Any line perpendicular to BC is $3x - 2y + \lambda = 0$
As it passes through the point $A(-3/5, 8/5)$



$\therefore \quad \dfrac{-9}{5} - \dfrac{16}{5} + \lambda = 0 \Rightarrow \lambda = 5$
$\therefore$ Equation of altitude AD is
$3x - 2y + 5 = 0 \qquad$ ...(i)
Any line perpendicular to side AC is $x + 4y + \mu = 0$
As it passes through the point $B(-3, 4)$
$\therefore \quad -3 + 16 + \mu = 0 \Rightarrow \mu = -13$
$\therefore$ Equation of altitude BE is $x + 4y - 13 = 0 \qquad$ ...(ii)
Now orthocentre of a triangle is the point of intersection of the altitudes of the triangle.
On solving the equation (i) of AD and (ii) of BE, we get $x = 3/7, y = 22/7$
$\therefore$ orthocentre = $(3/7, 22/7)$
As both the co-ordinates are positive, orthocentre lies in the first quadrant.

17. $|AP - BP| = 6$

We know that locus of a point, difference of whose distances from two fixed points is constant, is a hyperbola with the fixed points as focii and the difference of distances as length of transverse axis.

$\because$ A = (0, 4) and B = (0, –4)

$\therefore$ $ae = 4$ and $2a = 6 \Rightarrow a = 3, e = 4/3$

$\Rightarrow b^2 = 9\left(\frac{16}{9} - 1\right) = 7$

$\because$ foci being on *y*-axis, it is vertical hyperbola

$\therefore$ Equation of the hyperbola is $\frac{y^2}{9} - \frac{x^2}{7} = 1$

18. Given curve : $|x| + |y| = 1$

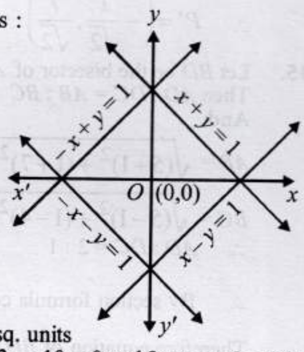
This curve represents four lines :

$x + y = 1, x - y = 1, -x + y = 1$

and $-x - y = 1$

These enclose a square of side = Distance between opposite sides $x + y = 1$ and $x + y = -1$

$\therefore$ Side $= \frac{1+1}{\sqrt{1+1}} = \sqrt{2}$



$\therefore$ Required area = $(\text{side})^2$ = 2 sq. units

19. **(True)** Intersection point of $x + 2y - 10 = 0$ and $2x + y + 5 = 0$ is $\left(\frac{-20}{3}, \frac{25}{3}\right)$ which clearly satisfies the line $5x + 4y = 0$. Hence the given statement is true.

20. **(d)** Slope of $x + 3y = 4$ is $-1/3$ and slope of $6x - 2y = 7$ is 3. Since, product of the two slopes is -1, which shows that both diagonals are perpendicular. Hence *PQRS* must be a rhombus.

21. **(c)** *PQRS* will represent a parallelogram if and only if the mid-point of *PR* is same as that of the mid-point of *QS*. *i.e.,* if and only if

$\frac{1+5}{2} = \frac{4+a}{2}$ and $\frac{2+7}{2} = \frac{6+b}{2} \Rightarrow a = 2$ and $b = 3$.

22. **(a, c)** Substituting the co-ordinates of the points (1, 3), (5, 0) and (– 1, 2) in $3x + 2y$, we obtain the value 8, 15 and 1 which are all +ve. Therefore, all the points lying inside the triangle formed by given points satisfy $3x + 2y \geq 0$.

$\therefore$ (a) is correct.

Substituting the co-ordinates of the given points in $2x + y - 13$, we find the values – 8, – 3 and – 13 which are all –ve.

$\therefore$ (b) is not correct.

Again substituting the given points in $2x - 3y - 12$ we get – 19, – 2, – 20 which are all –ve.

It follows that all points lying inside the triangle formed by given points satisfy $2x - 3y - 12 \leq 0$.

$\therefore$ (c) is correct.

Finally substituting the co-ordinates of the given points in $-2x + y$, we get 1, – 10 and 4 which are not all +ve.

$\therefore$ (d) is not correct.

Therefore, (a) and (c) are the correct answers.

23. **(e)** Let *A* (0, 8/3), *B* (1, 3) and *C* (82, 30).

Now, slope of line $AB = \frac{3 - 8/3}{1 - 0} = \frac{1}{3}$

Slope of line $BC = \frac{30-3}{82-1} = \frac{27}{81} = \frac{1}{3}$

$\Rightarrow$ *AB* || *BC* and *B* is common point.

$\therefore$ *A*, *B*, *C* are collinear.

24. Equation of the line passing through *P* (*h*, *k*) and parallel to *x*-axis is

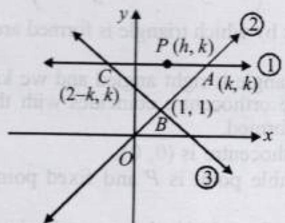
$y = k$. ... (i)

Other two given lines are

$y = x$ ... (ii)

and $x + y = 2$ ... (iii)

Let *ABC* be the Δ formed by the lines (i), (ii) and (iii), as shown in the figure.



On solving the three equations pairwise we get the co-ordinates of vertices *A*, *B* and *C* as *A* (*k*, *k*), *B* (1, 1) and *C* (2 – *k*, *k*)

$\therefore$ Area of $\Delta ABC = \frac{1}{2}|k(1 - k) + 1(k - k) + (2 - k)(k - 1)|$

$\Rightarrow (k - 1)^2 = 4h^2$

$\Rightarrow k - 1 = 2h$ or $k - 1 = -2h$

$\Rightarrow k = 2h + 1$ or $k = -2h + 1$

$\therefore$ Locus of (*h*, *k*) is, $y = 2x + 1$ or $y = -2x + 1$.

25. Let slope of the given line be *m*.

Then equation of the line is

$y - 2 = m(x - 8)$, where $m < 0$

$\Rightarrow P \equiv \left(8 - \frac{2}{m}, 0\right)$ and $Q \equiv (0, 2 - 8m)$

Now, $OP + OQ = \left|8 - \frac{2}{m}\right| + |2 - 8m|$

$= 10 + \frac{2}{-m} + 8(-m) \geq 10 + 2\sqrt{\frac{2}{-m} \times 8(-m)} \geq 18$

26. Let the co-ordinates of the vertices of the ΔABC be $A(a_1, b_1)$, $B(a_2, b_2)$ and $C(a_3, b_3)$ and co-ordinates of the vertices of the ΔPQR be $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$

Slope of $QR = \frac{y_2 - y_3}{x_2 - x_3}$

$\Rightarrow$ Slope of straight line perpendicular to $QR = -\frac{x_2 - x_3}{y_2 - y_3}$

Equation of straight line passing through $A(a_1, b_1)$ and perpendicular to *QR* is

$$y - b_1 = -\frac{x_2 - x_3}{y_2 - y_3}(x - a_1)$$

$\Rightarrow$ $(x_2 - x_3)x + (y_2 - y_3)y - a_1(x_2 - x_3) - b_1(y_2 - y_3) = 0$ ... (i)

Similarly equation of straight line from B and perpendicular to RP is

$(x_3 - x_1)x + (y_3 - y_1)y - a_2(x_3 - x_1) - b_2(y_3 - y_1) = 0$ ... (ii)

and equation of straight line from C and perpendicular to PQ is

$(x_1 - x_2)x + (y_1 - y_2)y - a_3(x_1 - x_2) - b_3(y_1 - y_2) = 0$ ... (iii)

As straight lines (i), (ii) and (iii) are given to be concurrent,

$$\therefore \begin{vmatrix} x_2 - x_3 & y_2 - y_3 & a_1(x_2 - x_3) + b_1(y_2 - y_3) \\ x_3 - x_1 & y_3 - y_1 & a_2(x_3 - x_1) + b_2(y_3 - y_1) \\ x_1 - x_2 & y_1 - y_2 & a_3(x_1 - x_2) + b_3(y_1 - y_2) \end{vmatrix} = 0 \text{ ... (iv)}$$

Operating $R_1 \to R_1 + R_2 + R_3$, we get

$$\begin{vmatrix} 0 & 0 & S \\ x_3 - x_1 & y_3 - y_1 & a_2(x_3 - x_1) + b_2(y_3 - y_1) \\ x_1 - x_2 & y_1 - y_2 & a_3(x_1 - x_2) + b_3(y_1 - y_2) \end{vmatrix} = 0,$$

where $[S = a_1(x_2 - x_3) + b_1(y_2 - y_3) + a_2(x_3 - x_1) + b_2(y_3 - y_1) + a_3(x_1 - x_2) + b_3(y_1 - y_2)]$

On expanding along R_1, we get

$\Rightarrow$ $[(x_3 - x_1)(y_1 - y_2) - (x_1 - x_2)(y_3 - y_1)]S = 0$

$$\Rightarrow \left[\frac{y_1 - y_2}{x_1 - x_2} - \frac{y_3 - y_1}{x_3 - x_1}\right]S = 0$$

$\Rightarrow$ $[m_{PQ} - m_{PR}]S = 0 \Rightarrow S = 0$

$[\because m_{PQ} = m_{PR} \Rightarrow PQ \parallel PR$

which is not possible in $\Delta PQR]$

$\Rightarrow$ $a_1(x_2 - x_3) + b_1(y_2 - y_3) + a_2(x_3 - x_1) + b_2(y_3 - y_1) + a_3(x_1 - x_2) + b_3(y_1 - y_2) = 0$ ... (v)

$\Rightarrow$ $x_1(a_3 - a_2) + y_1(b_3 - b_2) + x_2(a_1 - a_3) + y_2(b_1 - b_3) + x_3(a_2 - a_1) + y_3(b_2 - b_1) = 0$ ...(vi)

(Rearranging the equation (v))

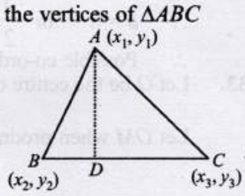
But above condition shows

$$\begin{vmatrix} a_3 - a_2 & b_3 - b_2 & x_1(a_3 - a_2) + y_1(b_3 - b_2) \\ a_1 - a_3 & b_1 - b_3 & x_2(a_1 - a_3) + y_2(b_1 - b_3) \\ a_2 - a_1 & b_2 - b_1 & x_3(a_2 - a_1) + y_3(b_2 - b_1) \end{vmatrix} = 0 \text{ ...(vii)}$$

[Using the fact that as (iv) $\Leftrightarrow$ (v) in the same way (vi) $\Leftrightarrow$ (vii)]

Clearly equation (vii) shows that lines through P and perpendicular to BC, through Q and perpendicular to AB are concurrent.

27. Let $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ be the vertices of ΔABC



Then equation of alt. AD is

$$y - y_1 = -\left[\frac{x_2 - x_3}{y_2 - y_3}\right](x - x_1)$$

$\Rightarrow$ $(x - x_1)(x_2 - x_3) + (y - y_1)(y_2 - y_3) = 0$ ... (i)

Similarly equations of other two attitudes are

$(x - x_2)(x_3 - x_1) + (y - y_2)(y_3 - y_1) = 0$ ...(ii) and

$(x - x_3)(x_1 - x_2) + (y - y_3)(y_1 - y_2) = 0$ ...(iii)

Now, above three lines will be concurrent if

$$\begin{vmatrix} x_2 - x_3 & y_2 - y_3 & -x_1(x_2 - x_3) - y_1(y_2 - y_3) \\ x_3 - x_1 & y_3 - y_1 & -x_2(x_3 - x_1) - y_2(y_3 - y_1) \\ x_1 - x_2 & y_1 - y_2 & -x_3(x_1 - x_2) - y_3(y_1 - y_2) \end{vmatrix} = 0$$

On applying operation $R_1 \to R_1 + R_2 + R_3$ in L.H.S., each element of R_1 becomes 0.

$\therefore$ Value of determinant = 0 = R.H.S.

Therefore, the altitudes are concurrent.

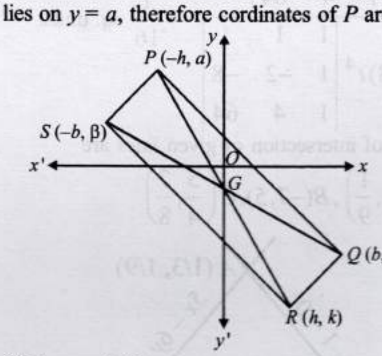
28. Let the co-ordinates of Q be (b, α) and that of S be $(-b, \beta)$. Let PR and SQ intersect each other at G.

Let the co-ordinates of R be (h, k).

$\therefore$ The x-coordinates of P is $-h$

($\because$ G is the mid point of PR)

As P lies on $y = a$, therefore cordinates of P are $(-h, a)$.



$\because$ PQ is parallel to $y = mx$,

$\therefore$ Slope of $PQ = m$

$$\Rightarrow \frac{\alpha - a}{b + h} = m \Rightarrow \alpha = a + m(b + h) \quad \text{... (i)}$$

Also $RQ \perp PQ \Rightarrow$ Slope of $RQ = \frac{-1}{m}$

$$\Rightarrow \frac{k - \alpha}{h - b} = \frac{-1}{m} \Rightarrow \alpha = k + \frac{1}{m}(h - b) \quad \text{... (ii)}$$

From (i) and (ii), we get

$$a + m(b + h) = k + \frac{1}{m}(h - b)$$

$\Rightarrow$ $(m^2 - 1)h - mk + b(m^2 + 1) + am = 0$

$\therefore$ Locus of vertex $R(h, k)$ is

$(m^2 - 1)x - my + b(m^2 + 1) + am = 0.$

29. Given curve : $y = x^3$ ... (i)

Let the point, P_1 be (t, t^3), $t \neq 0$

Then slope of tangent at $P_1 = \frac{dy}{dx} = (3x^2)_{x=t} = 3t^2$

$\therefore$ Equation of tangent at P_1 is

$y - t^3 = 3t^2(x - t) \Rightarrow y = 3t^2x - 2t^3$

$\Rightarrow$ $3t^2x - y - 2t^3 = 0$ ... (ii)

Now this tangent meets the curve again at P_2 which can be obtained by solving (i) and (ii)

i.e., $3t^2x - x^3 - 2t^3 = 0 \Rightarrow x^3 - 3t^2x + 2t^3 = 0$

$\Rightarrow$ $(x - t)^2(x + 2t) = 0 \Rightarrow x = -2t$ as $x = t$ is for P_1

$\therefore$ $y = -8t^3$

Hence point P_2 is $(-2t, -8t^3)$

Similarly, we can find that tangent at P_2 which meets the curve again at P_3 $(4t, 64t^3)$.

Similarly, $P_4 \equiv (-8t, -512t^3)$ and so on.

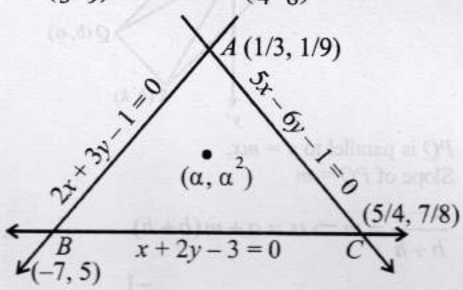
We observe that abscissa of points $P_1, P_2, P_3 \ldots$ are $t, -2t, 4t, \ldots$ which form a GP with common ratio -2.

Now, $$\frac{ar(\Delta P_1P_2P_3)}{ar(\Delta P_2P_3P_4)} = \frac{\begin{vmatrix} 1 & t & t^3 \\ 1 & -2t & -8t^3 \\ 1 & 4t & 64t^3 \end{vmatrix}}{\begin{vmatrix} 1 & -2t & -8t^3 \\ 1 & 4t & 64t^3 \\ 1 & -8t & -512t^3 \end{vmatrix}}$$

$$= \frac{t^4 \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & -8 \\ 1 & 4 & 64 \end{vmatrix}}{(-2)(-8)t^4 \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & -8 \\ 1 & 4 & 64 \end{vmatrix}} = \frac{1}{16} \text{ sq. units.}$$

30. The points of intersection of given lines are

$$A\left(\frac{1}{3}, \frac{1}{9}\right), B(-7, 5), C\left(\frac{5}{4}, \frac{7}{8}\right)$$



If (α, α^2) lies inside the triangle formed by the given lines, then $\left(\frac{1}{3}, \frac{1}{9}\right)$ and (α, α^2) lie on the same side of the line $x + 2y - 3 = 0$

$$\Rightarrow \frac{\alpha + 2\alpha^2 - 3}{\frac{1}{3} + \frac{2}{9} - 3} > 0 \Rightarrow 2\alpha^2 + \alpha - 3 > 0 \quad \ldots \text{(i)}$$

Similarly $\left(\frac{5}{4}, \frac{7}{8}\right)$ and (α, α^2) lie on the same side of the line $2x + 3y - 1 = 0$.

$$\Rightarrow \frac{2\alpha + 3\alpha^2 - 1}{\frac{10}{4} + \frac{21}{8} - 1} > 0 \Rightarrow 3\alpha^2 + 2\alpha - 1 > 0 \quad \ldots \text{(ii)}$$

$(-7, 5)$ and (α, α^2) lie on the same side of the line $5x - 6y - 1 = 0$.

$$\Rightarrow \frac{5\alpha + 6\alpha^2 - 1}{-35 - 30 - 1} > 0 \Rightarrow 6\alpha^2 - 5\alpha + 1 > 0 \quad \ldots \text{(iii)}$$

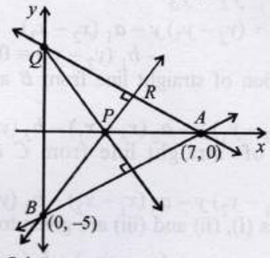
Now common solution of (i), (ii) and (iii) is obtained as

$$\alpha \in \left(-\frac{3}{2}, -1\right) \cup \left(\frac{1}{2}, 1\right)$$

31. Equation of the line AB is

$$\frac{x}{7} - \frac{y}{5} = 1 \Rightarrow 5x - 7y - 35 = 0$$

Equation of line $PQ \perp AB$ is $7x + 5y + \lambda = 0$ which meets x and y axis at points $P(-\lambda/7, 0)$ and $Q(0, -\lambda/5)$ respectively.



Equation of AQ is

$$\frac{x}{7} + \frac{y}{-\lambda/5} = 1 \Rightarrow \lambda x - 35y - 7\lambda = 0 \quad \ldots\ldots\ldots \text{(i)}$$

Equation of BP is

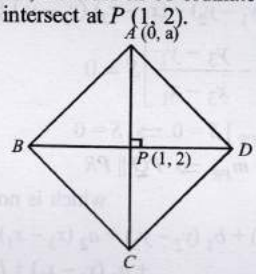
$$\frac{-7x}{\lambda} - \frac{y}{5} = 1 \Rightarrow 35x + \lambda y + 5\lambda = 0 \quad \ldots\ldots\ldots \text{(ii)}$$

Locus of R the point of intersection of (i) and (ii) can be obtained by eliminating λ from these equations as follows

$$35x + (5 + y)\left(\frac{35y}{x - 7}\right) = 0$$

$$\Rightarrow 35x(x - 7) + 35y(5 + y) = 0 \Rightarrow x^2 + y^2 - 7x + 5y = 0$$

32. A being on y-axis, consider its co-ordinates as $(0, a)$. The diagonals intersect at $P(1, 2)$.



Again we know that diagonals will be parallel to the angle bisectors of the two lines $y = x + 2$ and $y = 7x + 3$

i.e., $$\frac{x - y + 2}{\sqrt{2}} = \pm \frac{7x - y + 3}{5\sqrt{2}}$$

$\Rightarrow$ $5x - 5y + 10 = \pm(7x - y + 3)$

$\Rightarrow$ $2x + 4y - 7 = 0$ and $12x - 6y + 13 = 0$

Slope of $2x + 4y - 7 = 0$ is $m_1 = -1/2$ and slope of $12x - 6y + 13 = 0$ is $m_2 = 2$

Let diagonal d_1 be parallel to $2x + 4y - 7 = 0$ and diagonal d_2 be parallel to $12x - 6y + 13 = 0$. The vertex A could be on any of the two diagonals. Hence slope of AP is either $-1/2$ or 2.

$$\Rightarrow \frac{2 - a}{1 - 0} = 2 \text{ or } \frac{-1}{2}$$

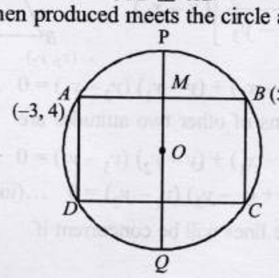
$$\therefore \quad a = 0 \text{ or } \frac{5}{2}$$

$\therefore$ Possible co-ordinates of A is $(0, 0)$ or $(0, 5/2)$

33. Let O be the centre of the circle. M is the mid point of AB. Then

$$OM \perp AB$$

Let OM when produced meets the circle at P and Q.



$\therefore$ PQ is a diameter perpendicular to AB and passing through M.

$$M = \left(\frac{-3+5}{2}, \frac{4+4}{2}\right) = (1,4)$$

Slope of $AB = \frac{4-4}{5+3} = 0$

$\therefore$ PQ, being perpendicular to AB, is a line parallel to y-axis passing through (1, 4).

$\therefore$ Its equation is

$x = 1$ ...... (i)

Also eq. of one of the diameter given is

$4y = x + 7$ ...... (ii)

On solving (i) and (ii), we get co-ordinates of centre O as (1, 2)

Let co-ordinates of D be (α, β)

Since O is mid point of BD,

$\therefore \left(\frac{\alpha+5}{2}, \frac{\beta+4}{2}\right) = (1,2) \Rightarrow \alpha = -3, \beta = 0$

$\therefore$ Co-ordinate of $D = (-3, 0)$

Now $AD = \sqrt{(-3+3)^2 + (4-0)^2} = 4$

and $AB = \sqrt{(5+3)^2 + (4-4)^2} = 8$

$\therefore$ Area of rectangle $ABCD = AB \times AD = 8 \times 4$
$= 32$ square units.

34. Area of $\Delta ABC = \frac{1}{2}[6(7) + 3(5) + 4(-2)] = \frac{49}{2}$

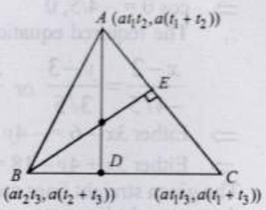
Area of $\Delta PBC = \frac{1}{2}(7x + 7y - 14) = \frac{7}{2}|x + y - 2|$

Now, $\frac{ar(\Delta PBC)}{ar(\Delta ABC)} = \frac{\frac{7}{2}|x+y-2|}{\frac{49}{2}} = \left|\frac{x+y-2}{7}\right|$

35. Slope of BC

$$= \frac{a(t_1+t_3) - a(t_2+t_3)}{at_1t_3 - at_2t_3}$$

$$= \frac{a(t_1+t_3-t_2-t_3)}{at_3(t_1-t_2)} = \frac{1}{t_3}$$



$\therefore$ Slope of $AD = -t_3$

$\therefore$ Equation of AD,

$y - a(t_1 + t_2) = -t_3(x - at_1t_2)$

$\Rightarrow xt_3 + y = at_1t_2t_3 + a(t_1 + t_2)$ .... (i)

Similarly, by symm. equation of BE is

$\Rightarrow xt_1 + y = at_1t_2t_3 + a(t_2 + t_3)$ .... (ii)

On solving (i) and (ii), we get

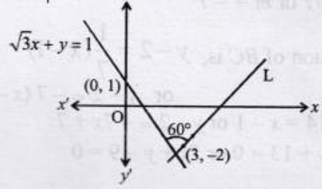
$x = -a$ and $y = a(t_1 + t_2 + t_3) + at_1t_2t_3)$

$\therefore$ Orthocentre is $H(-a, a(t_1 + t_2 + t_3) + at_1t_2t_3)$

Topic-2: Various Forms of Equation of a Line

1. **(b)** Let the slope of line L be m. Then

$$\left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \sqrt{3}$$



$\Rightarrow m + \sqrt{3} = \pm(\sqrt{3} - 3m)$

$\Rightarrow 4m = 0$ or $2m = 2\sqrt{3} \Rightarrow m = 0$ or $m = \sqrt{3}$

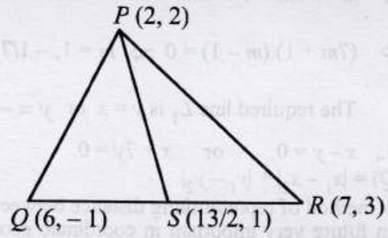
$\because$ L intersects x-axis, $\therefore m = \sqrt{3}$

$\therefore$ Equation of L is $y + 2 = \sqrt{3}(x - 3)$

$\Rightarrow \sqrt{3}x - y - (2 + 3\sqrt{3}) = 0$

2. **(d)** S is the midpoint of Q and R

$\therefore S \equiv \left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$



Now slope of $PS = \frac{2-1}{2-13/2} = -\frac{2}{9}$

Now equation of the line passing through (1, – 1) and parallel to PS is

$y + 1 = -\frac{2}{9}(x-1) \Rightarrow 2x + 9y + 7 = 0$

3. **(c)** $x^2 - 5x + 6 = 0$

$\Rightarrow (x-2)(x-3) = 0$

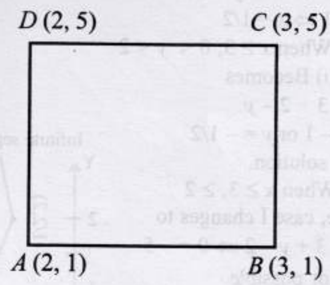
$\therefore x = 2$ and $x = 3$

And $y^2 - 6y + 5 = 0$

$\Rightarrow (y-1)(y-5) = 0$

$\therefore y = 1$ and $y = 5$

The sides of parallelogram are $x = 2, x = 3, y = 1, y = 5$.



$\therefore$ Diagonal AC is $\frac{y-1}{5-1} = \frac{x-2}{3-2} \Rightarrow y = 4x - 7$

Equation of diagonal BD is $\frac{x-2}{3-2} = \frac{y-5}{1-5} \Rightarrow 4x + y = 13$

4. **(b, c)** We know that length of intercept made by a circle on a line is given by $= 2\sqrt{r^2 - p^2}$, where

p = perpendicular distance of the line from the centre of the circle.

Here, circle is $x^2 + y^2 - x + 3y = 0$ with centre $\left(\frac{1}{2}, \frac{-3}{2}\right)$ and

radius $= \frac{\sqrt{10}}{2}$

Let $L_1 : y = mx$ (any line through origin)

Now, $L_2 : x + y - 1 = 0$ (given line)

ATQ circle makes equal intercepts on L_1 and L_2

$$\Rightarrow \quad 2\sqrt{\frac{10}{4}-\frac{\left(\frac{m}{2}+\frac{3}{2}\right)^2}{m^2+1}} = 2\sqrt{\frac{10}{4}-\frac{\left(\frac{1}{2}-\frac{3}{2}-1\right)^2}{2}}$$

$$\Rightarrow \quad \frac{\left(\frac{m+3}{2}\right)^2}{m^2+1} = 2$$

$\Rightarrow \quad m^2 + 6m + 9 = 8m^2 + 8 \quad \Rightarrow 7m^2 - 6m - 1 = 0$

$\Rightarrow \quad (7m + 1)(m - 1) = 0 \Rightarrow m = 1, -1/7$

$\therefore$ The required line L_1 is $y = x$ or $y = -\frac{x}{7}$,

i.e., $x - y = 0$ or $x + 7y = 0$.

5. $d : (P, Q) = |x_1 - x_2| + |y_1 - y_2|$.

It is new method of representinng distance between points P and Q and in future very important in coordinate geometry.

Now, let $P(x, y)$ be any point in the first quadrants have

$d(P, 0) = |x - 0| + |y - 0| = |x| + |y| = x + y$ [$\because$ x, y > 0]

$d(P, A) = |X - 3| + |Y - 2|$ [given]

$d(P, 0) = d(P, A)$ [given]

$\Rightarrow x + y = |x - 3| + |y - 2|$ ...(i)

Case I : When $0 < x < 3, 0 < y < 2$

In this case, Eq. (i) becomes

$x + y = 3 - x + 2 - y$

$\Rightarrow \quad 2x + 2y = 5$

or $\quad x + y = 5/2$

Case II : When $0 < x < 3, y \geq 2$

Now, Eq. (i) becomes

$x + y = 3 - x + y - 2$

$\Rightarrow \quad 2x = 1 \Rightarrow x = 1/2$

Case III : When $x \geq 3, 0 < y < 2$

Now, Eq. (i) Becomes

$x + y = x - 3 + 2 - y$

$\Rightarrow \quad 2y = -1$ or $y = -1/2$

Hence, no solution.

Case IV: When $x \geq 3, \geq 2$

In this case, case I changes to

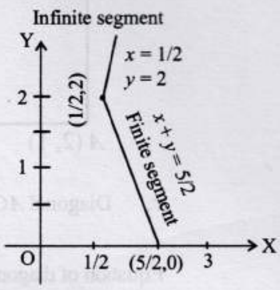
$x + y = x - 3 + y - 2 \Rightarrow 0 = -5$

which is not possible.

Hence, the solution set is

$\{(x, y)\} | x = \frac{1}{2}, y \geq | \cup \{(x, y)\}$

$x + y = 5/2, 0 < x < 3, 0 < y > 2\}$

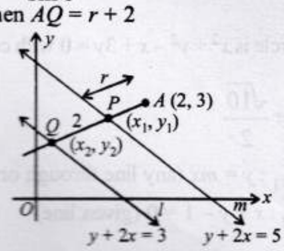


The graph is given in adjoining figure.

6. Let the equation of line through A which makes an intercept of 2 units between $2x + y = 3$ and $2x + y = 5$

be $\quad \frac{x-2}{\cos\theta} = \frac{y-3}{\sin\theta} = r$

Let $AP = r$ then $AQ = r + 2$



Then for point $P(x_1, y_1)$,

$$\frac{x_1-2}{\cos\theta} = \frac{y_1-3}{\sin\theta} = r \Rightarrow \frac{2(x_1-2)+(y_1-3)}{2\cos\theta+\sin\theta} = r$$

$$\left[\text{using } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{\lambda a_1 + \mu b_1}{\lambda a_2 + \mu b_2}\right]$$

$$\Rightarrow \frac{(2x_1+y_1)-7}{2\cos\theta+\sin\theta} = r \Rightarrow \frac{5-7}{2\cos\theta+\sin\theta} = r$$

[using $2x_1 + y_1 = 5$ as $P(x_1, y_1)$ lies on $2x + y = 5$]

$$\frac{-2}{2\cos\theta+\sin\theta} = r \quad \text{... (i)}$$

For pt $Q(x_2, y_2)$,

$$\frac{x_2-2}{\cos\theta} = \frac{y_2-3}{\sin\theta} = r+2$$

$$\Rightarrow \frac{2(x_2-2)+(y_2-3)}{2\cos\theta+\sin\theta} = r+2$$

$$\Rightarrow \frac{-4}{2\cos\theta+\sin\theta} = r+2 \quad \text{... (ii)}$$

[using $y_2 + 2x_2 = 3$ as Q lies on $y + 2x = 3$]

On subtracting (i) from (ii),

$$\frac{-2}{2\cos\theta+\sin\theta} = 2$$

$\Rightarrow \quad 2\cos\theta + \sin\theta = -1$ ... (iii)

$\Rightarrow \quad 2\cos\theta = -(1 + \sin\theta)$

Squaring on both sides, we get

$\Rightarrow \quad 4\cos^2\theta = 1 + 2\sin\theta + \sin^2\theta$

$\Rightarrow \quad (5\sin\theta - 3)(\sin\theta + 1) = 0 \Rightarrow \sin\theta = 3/5, -1$

$\Rightarrow \quad \cos\theta = -4/5, 0$ [using eq. (iii)]

$\therefore$ The required equation is either

$$\frac{x-2}{-4/5} = \frac{y-3}{3/5} \text{ or } \frac{x-2}{0} = \frac{y-3}{-1}$$

$\Rightarrow$ Either $3x - 6 = -4y + 12$ or $x - 2 = 0$

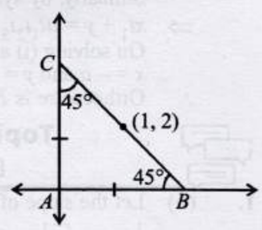
$\Rightarrow$ Either $3x + 4y - 18 = 0$ or $x - 2 = 0$

7. The given straight lines are $3x + 4y = 5$ and $4x - 3y = 15$. Clearly these straight lines are perpendicular to each other as $m_1 m_2 = -1$ i.e., product of their slopes is -1. The given two lines intersect at A.

$\because \quad AB = AC$

$\therefore \angle B = \angle C = 45°$

Let slope of BC be m. Then



$$\tan 45° = \left|\frac{m+3/4}{1-\frac{3}{4}m}\right|$$

$\Rightarrow \quad 4m + 3 = \pm(4 - 3m)$

$\Rightarrow \quad 4m + 3 = 4 - 3m$ or $4m + 3 = -4 + 3m$

$\Rightarrow \quad m = 1/7$ or $m = -7$

$\therefore$ Equation of BC is, $y - 2 = \frac{1}{7}(x - 1)$

or $\quad y - 2 = -7(x - 1)$

$\Rightarrow \quad 7y - 14 = x - 1$ or $y - 2 = -7x + 7$

$\Rightarrow \quad x - 7y + 13 = 0$ or $7x + y - 9 = 0$

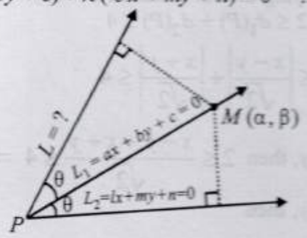
8. Let the equation of other line L, which passes through the point of intersection P of lines

$L_1 \equiv ax + by + c = 0$ .... (i)

and $L_2 \equiv \ell x + my + n = 0$ .... (ii)

be $L_1 + \lambda L_2 = 0$

$\Rightarrow (ax + by + c) + \lambda(\ell x + my + n) = 0$ .... (iii)



From figure it is clear that L_1 is the bisector of the angle between the lines given by (ii) and (iii) [i.e. L_2 and L]

Let $M(\alpha, \beta)$ be any point on L_1 then

$a\alpha + b\beta + c = 0$ ...... (iv)

Also from M, lengths of perpendiculars to lines L and L_2 given by equations (iii) and (iv), are equal.

$$\therefore \frac{\ell\alpha + m\beta + n}{\sqrt{\ell^2 + m^2}} = \pm \frac{(a\alpha + b\beta + c) + \lambda(l\alpha + m\beta + n)}{\sqrt{(a + \lambda l)^2 + (b + \lambda m)^2}}$$

$$\Rightarrow \frac{1}{\sqrt{\ell^2 + m^2}} = \pm \frac{\lambda}{\sqrt{(\ell^2 + m^2)\lambda^2 + 2(a\ell + bm)\lambda + (a^2 + b^2)}}$$ [using (iv)]

$\Rightarrow (\ell^2 + m^2)\lambda^2 + 2(a\ell + bm)\lambda + (a^2 + b^2) = \lambda^2(\ell^2 + m^2)$

$$\Rightarrow \lambda = -\frac{a^2 + b^2}{2(a\ell + bm)}$$

On substituting this value of λ in eq. (iii), we get the equation of L as

$$(ax + by + c) - \frac{(a^2 + b^2)}{2(a\ell + bm)}(\ell x + my + n) = 0$$

$\Rightarrow (a^2 + b^2)(\ell x + my + n) - 2(a\ell + bm)(ax + by + c) = 0$

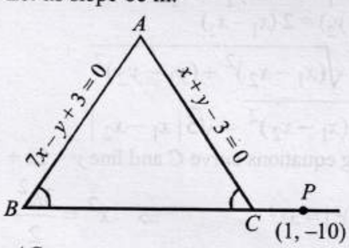
9. Let equations of equal sides AB and AC of isosceles ΔABC are

$7x - y + 3 = 0$

and $x + y - 3 = 0$

Now slope of $AB = 7$ and slope of $AC = -1$

The third side BC of the triangle passes through the point $(1, -10)$. Let its slope be m.



As $AB = AC$

$\therefore \angle B = \angle C$

$\Rightarrow \tan B = \tan C$

$$\therefore \left|\frac{7 - m}{1 + 7m}\right| = \left|\frac{-1 - m}{1 - m}\right|$$

$$\Rightarrow \frac{7 - m}{1 + 7m} = \pm\left(\frac{-1 - m}{1 - m}\right)$$

On taking '+' sign, we get

$(7 - m)(1 - m) = -(1 + m)(1 + 7m)$

$\Rightarrow 7 - 8m + m^2 + 7m^2 + 8m + 1 = 0$

$\Rightarrow 8m^2 + 8 = 0 \Rightarrow m^2 + 1 = 0$

It has no real solution.

On taking '–' sign, we get

$(7 - m)(1 - m) = (1 + m)(1 + 7m)$

$\Rightarrow 7 - 8m + m^2 - 7m^2 - 8m - 1 = 0$

$\Rightarrow -6m^2 - 16m + 6 = 0 \Rightarrow 3m^2 + 8m - 3 = 0$

$\Rightarrow (3m - 1)(m + 3) = 0 \Rightarrow m = 1/3, -3$

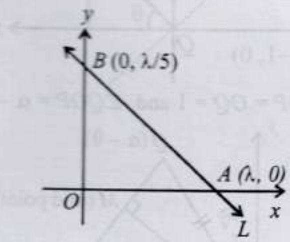
$\therefore$ The required line is

$$y + 10 = \frac{1}{3}(x - 1) \text{ or } y + 10 = -3(x - 1)$$

i.e. $x - 3y - 31 = 0$ or $3x + y + 7 = 0$.

10. The given line is $5x - y = 1$

$\therefore$ The equation of line L which is perpendicular to the given line is $x + 5y = \lambda$. This line meets co-ordinate axes at $A(\lambda, 0)$ and $B(0, \lambda/5)$.



Now, area of $\Delta OAB = \frac{1}{2} \times OA \times OB$

$$\Rightarrow 5 = \frac{1}{2} \times \lambda \times \frac{\lambda}{5} \Rightarrow \lambda^2 = 5^2 \times 2 \Rightarrow \lambda = \pm 5\sqrt{2}$$

$\therefore$ The equation of line L is $x + 5y - 5\sqrt{2} = 0$

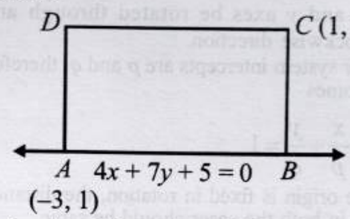
or $x + 5y + 5\sqrt{2} = 0$.

11. Let side AB of rectangle $ABCD$ lies along

$4x + 7y + 5 = 0$. .... (i)

As $(-3, 1)$ lies on the line, let it be vertex A.

Since $(1, 1)$ does not satisfy equation (i), therefore $(1, 1)$ is either vertex C or D.



If $(1, 1)$ is vertex D then slope of $AD = 0$

$\therefore$ AD is not perpendicular to AB, which contradict '$ABCD$ is a rectangle'.

$\therefore$ $(1, 1)$ are the co-ordinates of vertex C.

CD is a line parallel to AB and passing through C, therefore equation of CD is

$$y - 1 = -\frac{4}{7}(x - 1) \Rightarrow 4x + 7y - 11 = 0$$

Also BC is a line perpendicular to AB and passing through C, therefore equation of BC is

$$y-1=\frac{7}{4}(x-1)\Rightarrow 7x-4y-3=0$$

Similarly, AD is a line perpendicular to AB and passing through A $(-3, 1)$, therefore equation of line AD is

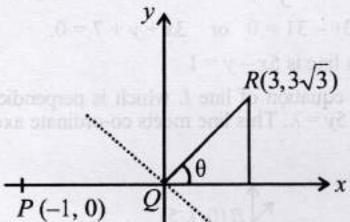
$$y-1=\frac{7}{4}(x+3)\Rightarrow 7x-4y+25=0$$

Topic-3: Distance Between two Lines, Angle Between two Lines and Bisector of the Angle Between the two Lines

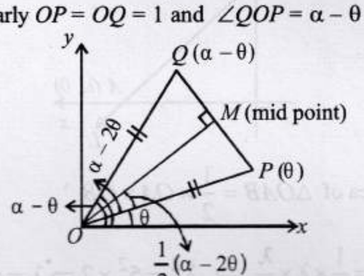
1. **(c)** $\tan\theta=\sqrt{3}\Rightarrow\theta=60^\circ\Rightarrow\angle PQR=120^\circ$

$\therefore$ Slope of bisector of $\angle PQR$ = tan 120°

Hence, equation of bisector is $\sqrt{3}x+y=0$



2. **(d)** Clearly $OP = OQ = 1$ and $\angle QOP = \alpha-\theta-\theta=\alpha-2\theta$.



The bisector of $\angle QOP$ will be a perpendicular bisector of PQ also. Hence Q is reflection of P in the line OM which makes an angle $\angle MOP+\angle POX$ with x-axis, i.e., $\frac{1}{2}(\alpha-2\theta)+\theta=\alpha/2$.

So that slope of OM is tan $\alpha/2$.

3. **(b)** As L has intercepts a and b on axes, equation of L is

$$\frac{x}{a}+\frac{y}{b}=1 \quad \text{........ (i)}$$

Let x and y axes be rotated through an angle θ in anticlockwise direction.

In new system intercepts are p and q, therefore equation of L becomes

$$\frac{x}{p}+\frac{y}{q}=1 \quad \text{........ (ii)}$$

As the origin is fixed in rotation, the distance of line from origin in both the cases should be same.

$$d=\left|\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right|=\left|\frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}}\right|\Rightarrow\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$$

$\therefore$ (b) is the correct option.

4. **(a)** Let $(-a,-b)$, $(0, 0)$, (a, b) and (a^2, ab) are the coordinates of A, B, C and D respectively.

Now, slope of $AB=\frac{b}{a}$ = Slope of BC = Slope of BD

$\therefore$ A, B, C, D are collinear.

5. **(6)** Let the point P be (x, y).

Then $d_1(P)=\left|\frac{x-y}{\sqrt{2}}\right|$ and $d_2(P)=\left|\frac{x+y}{\sqrt{2}}\right|$

For P lying in first quadrant $x>0$, $y>0$.

Now $2\le d_1(P)+d_2(P)\le 4$

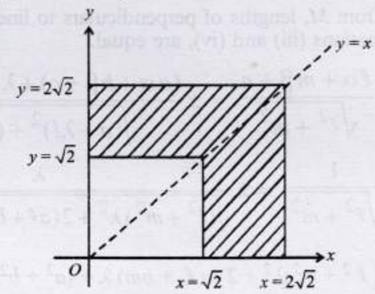
$$\Rightarrow 2\le\left|\frac{x-y}{\sqrt{2}}\right|+\left|\frac{x+y}{\sqrt{2}}\right|\le 4$$

If $x>y$, then $2\le\frac{x-y+x+y}{\sqrt{2}}\le 4\Rightarrow\sqrt{2}\le x\le 2\sqrt{2}$

If $x<y$, then

$$2\le\frac{y-x+x+y}{\sqrt{2}}\le 4 \text{ or } \sqrt{2}\le y\le 2\sqrt{2}$$

The required region is the shaded region in the figure given below.



$\therefore$ Required area $=\left(2\sqrt{2}\right)^2-\left(\sqrt{2}\right)^2=8-2=6$ sq units.

6. **(9)** Let locus point $P(x, y)$.

$\therefore$ According to question,

$$\left|\frac{\sqrt{2}x+y-1}{\sqrt{3}}\right|\left|\frac{\sqrt{2}x-y+1}{\sqrt{3}}\right|=\lambda^2$$

$$\Rightarrow\left|\frac{2x^2-(y-1)^2}{3}\right|=\lambda^2$$

So, $C:|2x^2-(y-1)^2|=3\lambda^2$

Let the line $y = 2x + 1$ meets C at two points $R(x_1, y_1)$ and $S(x_2, y_2)$

$\Rightarrow$ $y_1=2x_1+1$ and $y_2=2x_2+1$ ...(i)

$\Rightarrow$ $(y_1-y_2)=2(x_1-x_2)$

$\therefore$ $RS=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

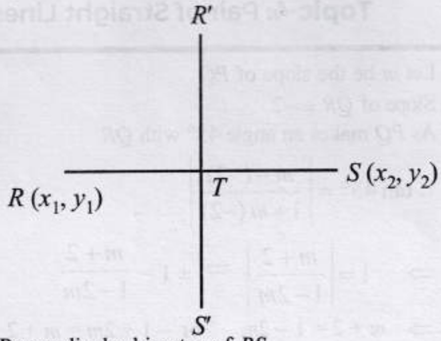
$RS=\sqrt{5(x_1-x_2)^2}=\sqrt{5}|x_1-x_2|$

On solving equations curve C and line $y = 2x + 1$, we get

$|2x^2-(2x)^2|=3\lambda^2 \quad\Rightarrow\quad x^2=\frac{3\lambda^2}{2}$

$\therefore RS=\sqrt{5}\left|\frac{2\sqrt{3}\lambda}{\sqrt{2}}\right|=\sqrt{30}\,\lambda=\sqrt{270}\Rightarrow 30\lambda^2=270\Rightarrow\lambda^2=9$

7. (77.14)



Perpendicular bisector of RS

$$T \equiv \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Since, $x_1 + x_2 = 0$ $\Rightarrow$ $y_1 + y_2 = 2$

So, $T = (0, 1)$ [from (i)]

Equation of $R'S'$:

$$(y-1) = -\frac{1}{2}(x-0) \Rightarrow x + 2y = 2$$

Let $R'(a_1, b_1)$ and $S'(a_2, b_2)$

$\therefore$ $D = (a_1 - a_2)^2 + (b_1 - b_2)^2 = 5(b_1 - b_2)^2$

On solving $x + 2y = 2$ and $|2x^2 - (y-1)^2| = 3\lambda^2$, we get

$\Rightarrow$ $|8(y-1)^2 - (y-1)^2| = 3\lambda^2$

$$\Rightarrow (y-1)^2 = \left(\frac{\sqrt{3}\lambda}{\sqrt{7}}\right)^2$$

$$y - 1 = \pm\frac{\sqrt{3}\lambda}{\sqrt{7}}$$

$$\Rightarrow b_1 = 1 + \frac{\sqrt{3}\lambda}{\sqrt{7}}, b_2 = 1 - \frac{\sqrt{3}\lambda}{\sqrt{7}}$$

$$D = 5\left(\frac{2\sqrt{3}\lambda}{\sqrt{7}}\right)^2 = \frac{5\times4\times3\lambda^2}{7} = \frac{5\times4\times27}{7} = 77.14$$

8. Let the variable line be $ax + by + c = 0$ .... (i)

$\therefore$ perpendicular distance of line from $(2, 0) = \frac{2a+c}{\sqrt{a^2+b^2}}$

Perpendicular distance of line from $(0, 2) = \frac{2b+c}{\sqrt{a^2+b^2}}$

Perpendicular distance of line from $(1, 1) = \frac{a+b+c}{\sqrt{a^2+b^2}}$

$$\text{Now, } \frac{2a+c}{\sqrt{a^2+b^2}} + \frac{2b+c}{\sqrt{a^2+b^2}} + \frac{a+b+c}{\sqrt{a^2+b^2}} = 0$$

$$\Rightarrow \frac{2a+c+2b+c+a+b+c}{\sqrt{a^2+b^2}} = 0$$

$\Rightarrow$ $a + b + c = 0$ .... (ii)

From (i) and (ii), we can say variable line (i) passes through the fixed point (1, 1).

9. If a, b, c are in A.P. then

$a + c = 2b \Rightarrow a - 2b + c = 0$

$\Rightarrow$ $ax + by + c = 0$ passes through the point $(1, -2)$.

10. Given that $3a + 2b + 4c = 0 \Rightarrow \frac{3}{4}a + \frac{1}{2}b + c = 0$

$\Rightarrow$ The set of lines $ax + by + c = 0$ passes through the point (3/4, 1/2) and hence concurrent at $\left(\frac{3}{4}, \frac{1}{2}\right)$.

11. **(True)** Intersection point of $x + 2y - 10 = 0$ and $2x + y + 5 = 0$ is $\left(\frac{-20}{3}, \frac{25}{3}\right)$ which clearly satisfies the line $5x + 4y = 0$. Hence the given statement is true.

12. **(a)** The intersection point of two given lines is $\left(\frac{-c}{a+b}, \frac{-c}{a+b}\right)$

Now, distance between (1, 1) and $\left(\frac{-c}{a+b}, \frac{-c}{a+b}\right) < 2\sqrt{2}$

$$\Rightarrow 2\left(1 + \frac{c}{a+b}\right)^2 < 8 \Rightarrow 1 + \frac{c}{a+b} < 2$$

$\Rightarrow$ $a + b - c > 0$

13. **(a, c)**

For concurrency of three given lines,

$$\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

On applying $C_1 = C_1 + C_2 + C_3$

$$\begin{vmatrix} p+q+r & q & r \\ p+q+r & r & p \\ p+q+r & p & q \end{vmatrix} = 0$$

$$\Rightarrow (p+q+r)\begin{vmatrix} 1 & q & r \\ 1 & r & p \\ 1 & p & q \end{vmatrix} = 0$$

On applying $C_1 = C_1 - C_2$, $C_2 = C_2 - C_3$,

$$(p+q+r)\begin{vmatrix} 0 & q-r & r-p \\ 0 & r-p & p-q \\ 1 & p & q \end{vmatrix} = 0$$

$\Rightarrow$ $(p + q + r)(pq - q^2 - rp + rq - r^2 + pr + pr - p^2) = 0$

$\Rightarrow$ $(p + q + r)(p^2 + q^2 + r^2 - pq - pr - rq) = 0$

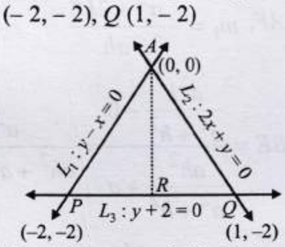
$\Rightarrow$ $p^3 + q^3 + r^3 - 3pqr = 0$

[$\because$ If $p + q + r = 0$, then $p^3 + q^3 + r^3 = 3pqr$]

It is clear that a, c are correct options.

14. **(c)** Point of intersection of L_1 and L_2 is $A(0, 0)$.

Also $P(-2, -2)$, $Q(1, -2)$



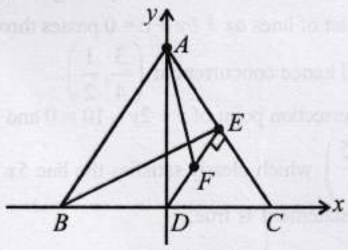
$\because$ AR is the bisector of $\angle PAQ$, therefore R divides PQ in the ratio of $AP : AQ$.

i.e., $PR : RQ = AP : AQ = 2\sqrt{2} : \sqrt{5}$

$\therefore$ Statement-1 is true.

Statement-2 is clearly false.

15. Consider BC as x-axis with origin at D i.e., the mid-point of BC and DA as y-axis.



$AB = AC$.

Let $BC = 2a$, then the co-ordinates of B and C are $(-a, 0)$ and $(a, 0)$ respectively.

Let $DA = h$, so that co-ordinates of A are $(0, h)$.

$\therefore$ Equation of AC is $\frac{x}{a} + \frac{y}{h} = 1$ .... (i)

And equation of $DE \perp$ to AC and passing through origin is

$\frac{x}{h} - \frac{y}{a} = 0 \Rightarrow x = \frac{hy}{a}$ .... (ii)

On solving (i) and (ii), we get the co-ordinates of point E as follows

$\frac{hy}{a^2} + \frac{y}{h} = 1 \Rightarrow h^2 y + a^2 y = a^2 h$

$\Rightarrow y = \frac{a^2 h}{a^2 + h^2} \Rightarrow x = \frac{ah^2}{a^2 + h^2}$

$\therefore$ co-ordinate of $E = \left(\frac{ah^2}{a^2 + h^2}, \frac{a^2 h}{a^2 + h^2}\right)$

Since F is mid pt. of DE, therefore, its co-ordinates

$= \left(\frac{ah^2}{2(a^2 + h^2)}, \frac{a^2 h}{2(a^2 + h^2)}\right)$

$\therefore$ Slope of $AF = \dfrac{h - \frac{a^2 h}{2(a^2 + h^2)}}{0 - \frac{ah^2}{2(a^2 + h^2)}} = \dfrac{2h(a^2 + h^2) - a^2 h}{-ah^2}$

$\Rightarrow$ Slope of AF, $m_1 = -\frac{a^2 + 2h^2}{ah}$ .... (iii)

And slope of $BE = \dfrac{\frac{a^2 h}{a^2 + h^2} - 0}{\frac{ah^2}{a^2 + h^2} + a} = \dfrac{a^2 h}{ah^2 + a^3 + ah^2}$

$\Rightarrow$ Slope of BF, $m_2 = \frac{ah}{a^2 + 2h^2}$ .... (iv)

From (iii) and (iv), we observe that

$m_1 m_2 = -1 \Rightarrow AF \perp BE$

Topic-4: Pair of Straight Lines

1. **(b)** Let m be the slope of PQ

Slope of $QR = -2$

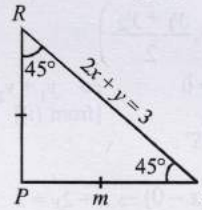
As PQ makes an angle 45° with QR

$\therefore \tan 45° = \left|\frac{m - (-2)}{1 + m(-2)}\right|$

$\Rightarrow 1 = \left|\frac{m + 2}{1 - 2m}\right| \Rightarrow \pm 1 = \frac{m + 2}{1 - 2m}$

$\Rightarrow m + 2 = 1 - 2m$ or $-1 + 2m = m + 2$

$\Rightarrow m = -1/3$ or $m = 3$



Since $PQ \perp PR$

$\therefore$ If slope of $PQ = -\frac{1}{3}$, then slope of $PR = 3$ and if slope of $PQ = 3$,

then slope of $PR = -\frac{1}{3}$

$\therefore$ Equation of PQ is $y - 1 = -\frac{1}{3}(x - 2)$

$\Rightarrow 3y - 3 = -x + 2 \Rightarrow x + 3y - 5 = 0$

and equation of PR is $3x - y - 5 = 0$

$\therefore$ Combined equation of PQ and PR is

$(x + 3y - 5)(3x - y - 5) = 0$

$\Rightarrow 3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

2. Given curve :

$3x^2 - y^2 - 2x + 4y = 0$ ... (i)

Let $y = mx + c$ be the chord of curve (i) which subtends a right angle at origin.

Then the combined eq. of lines joining points of intersection of curve (i) and chord $y = mx + c$ to the origin, can be obtained by making the equation of curve homogeneous with the help of equation of chord, as follows.

$3x^2 - y^2 - 2x\left(\frac{y - mx}{c}\right) + 4y\left(\frac{y - mx}{c}\right) = 0$

$\Rightarrow (3c + 2m)x^2 - 2(1 + 2m)xy + (4 - c)y^2 = 0$ ....(ii)

As a pair of lines represented by (ii) are perpendicular to each other, therefore we must have

Coeff. of x^2 + Coeff. of $y^2 = 0$

$\Rightarrow 3c + 2m + 4 - c = 0 \Rightarrow -2 = m \cdot 1 + c$

Which on comparing with equation $y = mx + c$ of chord, implies that $y = mx + c$ passes though $(1, -2)$.

$\therefore$ The family of chords must pass through $(1, -2)$.